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The temperature of non-spherical interstellar grains

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Abstract. A model of spheroidal particles is used to calculate the steady-state temperature of dust grains immersed in the interstellar radiation field. It is found that the temperature of non-spherical grains with the aspect ratios $a/b \lesssim 2$ deviates from that of spheres less than 10 %. More elongated or flattened particles are usually cooler than spheres of the same mass and in some cases the temperatures may differ by even about a factor of 2. The shape effects increase with the infrared absorptivity of the grain material and seem to be more important in dark interstellar clouds.

1. Introduction

The temperature of interstellar dust grains has been calculated by various authors many times, starting in the 1940s (see discussion in van de Hulst 1949). In the calculations one always assumed that the particles were spheres (see, e.g., Mathis et al. 1983). However, it is well known since the discovery of interstellar polarization (Hiltner 1949; Hall 1949; Dombrovski 1949) that the non-spherical grains should exist in the interstellar medium.

The first and single attempt to study the shape effects on grain temperature was made by Greenberg & Shah (1971). They considered metallic and dielectric Rayleigh spheroids and infinitely long icy cylinders of radius 0.1 μm. Their conclusion that non-spherical particles are about 10 % cooler than spheres is the result even included in textbooks (see Whittet 1992).

The calculation of the dust temperature is an essential step in any modelling of infrared (IR) emission from dust shells and discs, interstellar clouds, and galaxies. The dust temperature is included into the expressions for the determination of dust mass and cooling processes. The temperature of particles is also important for the process of molecule formation on grains.

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In this letter, we estimate the particle shape effects on the interstellar grain temperature. Calculations are made for compact homogeneous prolate and oblate spheroids of different semiaxes ratios, sizes and compositions.

2. Calculations

Let us consider an interstellar grain in thermal equilibrium with its surroundings. In the isotropic radiation field the grain temperature T_d can be obtained as a solution of the energy balance equation for absorbed and emitted energy [in $\mathrm{erg\,s^{-1}}$]

$$\int_0^\infty \overline{C}_{\mathrm{abs}}\, 4\pi J_\lambda^{\mathrm{ISRF}}\, \mathrm{d}\lambda = \int_0^\infty \overline{C}_{\mathrm{em}}\, 4\pi B_\lambda(T_d)\, \mathrm{d}\lambda\,, \qquad (1)$$

where $\overline{C}_{\mathrm{abs}}$ and $\overline{C}_{\mathrm{em}}$ are the absorption and emission cross-sections averaged over orientation and $4\pi J_\lambda^{\mathrm{ISRF}}$ the interstellar radiation field (ISRF) [in $\mathrm{erg\,cm^{-2}\,s^{-1}\,\mu m^{-1}}$].

We suppose that the grains are prolate or oblate homogeneous spheroids with the aspect ratio a/b (a and b are the major and minor semiaxes of a spheroid, respectively).

We characterize the particle size by the radius r_V of the sphere whose volume is equal to that of a spheroid. The major semiaxis of the spheroid is connected with r_V as follows:

$$a = r_V (a/b)^{2/3} \qquad (2)$$

for prolate spheroids and

$$a = r_V (a/b)^{1/3} \qquad (3)$$

for oblate ones. In our calculations, particles with sizes $r_V = 0.005 - 0.25\,\mu$m are considered.

Under interstellar conditions, we can generally assume that the incident radiation is non-polarized and the grains are arbitrarily oriented in space (3D-orientation). Then the mean absorption cross-sections can be found as

$$\overline{C}_{\mathrm{abs}} = \int_0^{\pi/2} \frac{1}{2}\,\left[Q_{\mathrm{abs}}^{\mathrm{TM}}(m_\lambda, r_V, \lambda, a/b, \alpha)+\right.$$
$$\left. Q_{\mathrm{abs}}^{\mathrm{TE}}(m_\lambda, r_V, \lambda, a/b, \alpha)\right] G(\alpha)\sin\alpha\,\mathrm{d}\alpha\,. \qquad (4)$$

Here, m_λ is the refractive index of the grain material, α the angle between the rotation axis of the spheroid and the wave-vector ($0° \leq \alpha \leq 90°$) and G the geometrical cross-section of a spheroid (the area of the particle shadow) which is

$$G(\alpha) = \pi r_{\mathrm{V}}^2 \left(\frac{a}{b}\right)^{-2/3} \left[\left(\frac{a}{b}\right)^2 \sin^2\alpha + \cos^2\alpha\right]^{1/2} \tag{5}$$

for a prolate spheroid and

$$G(\alpha) = \pi r_{\mathrm{V}}^2 \left(\frac{a}{b}\right)^{2/3} \left[\left(\frac{a}{b}\right)^{-2} \sin^2\alpha + \cos^2\alpha\right]^{1/2}. \tag{6}$$

for an oblate spheroid.

The energy emitted by a particle is proportional to its surface area. Then the emission cross-sections can be found as

$$\overline{C}_{\mathrm{em}} = S \int_0^{\pi/2} \frac{1}{2} \left[Q_{\mathrm{abs}}^{\mathrm{TM}}(m_\lambda, r_{\mathrm{V}}, \lambda, a/b, \alpha) + \right.$$
$$\left. Q_{\mathrm{abs}}^{\mathrm{TE}}(m_\lambda, r_{\mathrm{V}}, \lambda, a/b, \alpha)\right] \sin\alpha\, \mathrm{d}\alpha, \tag{7}$$

where

$$S = 2\pi r_{\mathrm{V}}^2 \left[\left(\frac{a}{b}\right)^{-2/3} + \left(\frac{a}{b}\right)^{1/3} \frac{\arcsin(e)}{e}\right] \tag{8}$$

for an prolate spheroid and

$$S = 2\pi r_{\mathrm{V}}^2 \left[\left(\frac{a}{b}\right)^{2/3} + \left(\frac{a}{b}\right)^{-4/3} \frac{\ln[(1+e)/(1-e)]}{2e}\right] \tag{9}$$

for an oblate spheroid and $e = \sqrt{1 - (a/b)^{-2}}$.

In Eqs. (4), (7), the superscripts TM and TE are related to two cases of the polarization of incident radiation (TM and TE modes). The efficiency factors $Q_{\mathrm{abs}}^{\mathrm{TM,TE}}$ are calculated from the solution to the light scattering problem for spheroids (see Voshchinnikov & Farafonov 1993 for details). The benchmark results given by Voshchinnikov et al. (1999) were used for a thorough testing of the numerical code.

The chemical composition of interstellar grains is a subject of continuing discussion. As usual, a mixture of carbon and silicate particles or composite grains are considered (see Henning 1998 for a recent review). We consider six species used earlier by Il'in & Voshchinnikov (1998) in the modelling of radiation pressure in envelopes of late-type giants. They are: an amorphous carbon (AC1), iron and magnetite as examples of highly absorbing materials; the astronomical silicate (astrosil), artificial dirty silicate (Ossenkopf et al. 1992; OHM-silicate) and clean glassy pyroxene as examples of different types of silicates. The choice of the optical constants of these materials is described by Il'in & Voshchinnikov (1998).[1] This sample was extended by two species: carbon material (cellulose) pyrolized at 1000° C (cel1000; Jäger et al. 1998) and dirty ice

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[1] The refractive indices also may be found in the database of optical constants (Henning et al. 1999).

used in the classical work of Greenberg & Shah (1971). In the last case, we take the imaginary part of the refractive index $k = 0.02$ in the wavelength range $\lambda\lambda = 0.17 - 1.2\,\mu\mathrm{m}$ as it was made by Greenberg (1968, 1971).

The interstellar radiation field is adopted according to Mathis et al. (1983) for the solar neighbourhood

$$4\pi J_\lambda^{\mathrm{ISRF}} = 4\pi J_\lambda^{\mathrm{UV}} + 4\pi \sum_{j=1}^3 W_j B_\lambda(T_j), \tag{10}$$

where the UV emission from early type stars $4\pi J_\lambda^{\mathrm{UV}}$ is given by Mezger et al. (1982). Other components of the ISRF are described by blackbody radiation with the temperatures $T_1 = 7500\,\mathrm{K}$, $T_2 = 4000\,\mathrm{K}$, $T_3 = 3000\,\mathrm{K}$, respectively. The corresponding dilution factors are $W_1 = 10^{-14}$, $W_2 = 10^{-13}$, $W_3 = 4\,10^{-13}$.

3. Results and discussion

We calculated the temperatures of prolate and oblate spheroidal particles with the aspect ratios $a/b = 1.1 - 10$ and compared the results with the temperatures of spherical particles of the same volume (or mass). The results are given in Table 1 for three basic materials (amorphous carbon, astrosil and ice) and $a/b = 4$. They are also shown in Figs. 1 and 2 for particles of single size $r_{\mathrm{V}} = 0.01\,\mu\mathrm{m}$.

As it is seen from Table 1, the size effects on the temperature for non-spherical particles are rather small as it is the case for spherical grains.
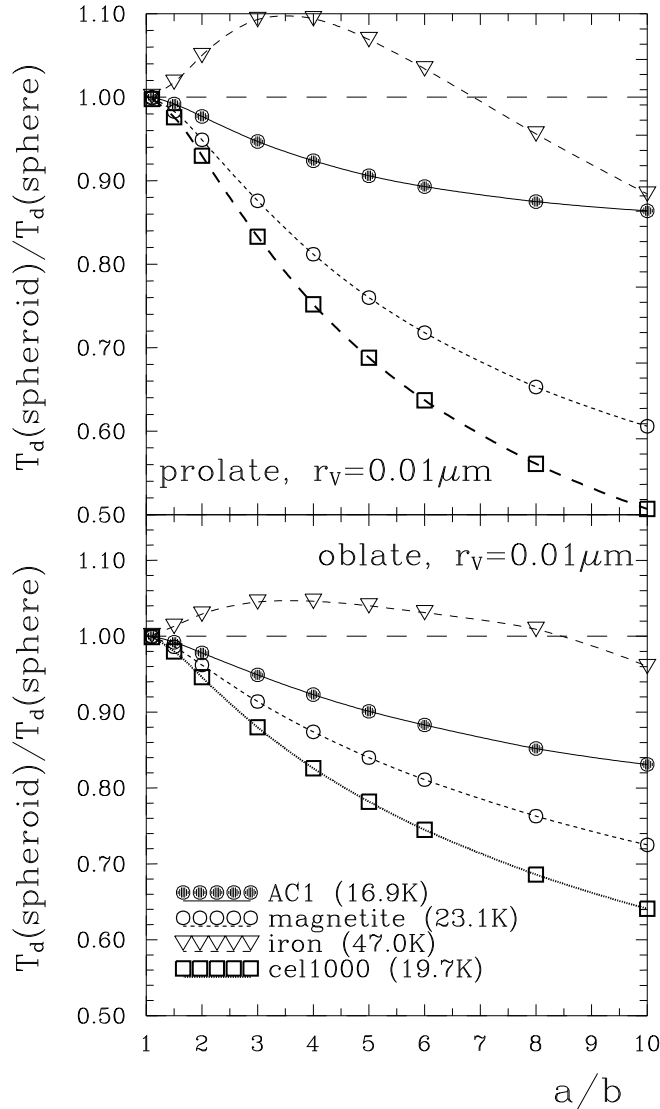
The shape effects are the most prominent in the case of grains consisting of absorbing materials (see Figs. 1, 2). Such particles emit more radiation at long wavelengths where the imaginary part of the refractive index k is large and increases significantly with wavelength (excluding AC1). In comparison with other materials presented in Fig. 1, the iron particles absorb much more radiation at visual wavelengths. For intermediate values of a/b, the emissive capacity of spheroids is almost the same as for spheres and therefore spheroids have larger temperatures. This explains the peculiar curves for iron in Fig. 1. Note also that for extremely prolate or oblate particles the decrease of the temperature with the growth a/b ceases. For example, for prolate particles from cel1000 the ratio $T_{\mathrm{d}}(\mathrm{spheroid})/T_{\mathrm{d}}(\mathrm{sphere}) = 0.51, 0.38, 0.28$ and 0.26 if $a/b = 10, 20, 50$ and 100, respectively.

The behaviour of k for all silicates is rather similar and the values of k in the IR are usually smaller than for materials with metallic properties. As a result, the ratio $T_{\mathrm{d}}(\mathrm{spheroid})/T_{\mathrm{d}}(\mathrm{sphere})$ changes in narrower limits in Fig. 2 in comparison with Fig. 1.

The behaviour similar to described above is kept when the incident radiation is slightly polarized ($\leq 10\,\%$) or its spectral distribution changes due to attenuation of the UV field and other components in Eq. (10). In both cases the relative changes of the ratio of temperatures remain in the limits smaller than $5\,\%$.

Table 1. The temperature of spherical and spheroidal ($a/b = 4$) grains in Kelvin

$r_{\rm V}, \mu$m	Amorphous carbon			Astronomical silicate			Ice		
	Sphere	Prolate	Oblate	Sphere	Prolate	Oblate	Sphere	Prolate	Oblate
0.005	16.8	15.6	15.6	15.1	13.4	13.6	15.2	14.7	14.4
0.010	16.9	15.6	15.6	15.3	13.5	13.7	15.2	14.7	14.4
0.020	17.1	15.7	15.6	15.3	13.4	13.5	15.2	14.6	14.3
0.030	17.1	15.7	15.6	15.1	13.2	13.3	14.9	14.4	14.2
0.050	17.2	15.6	15.5	14.7	12.9	13.0	14.5	14.1	13.8
0.100	17.4	15.6	15.5	14.4	12.5	12.2	14.0	13.6	13.4
0.150	17.4	15.4	15.3	14.4	12.1	12.3	13.9	13.4	13.2
0.250	16.9	14.8	14.8	14.4	12.3	12.3	13.7	12.8	12.6

Fig. 1. The shape dependence of the ratio of temperatures for prolate (upper panel) and oblate (lower panel) spheroids and spheres of materials with metallic properties. The grain composition and the temperature of spherical particles are indicated.

The alignment of dust grains does not affect strongly the temperature of non-spherical interstellar grains. Its influence becomes more important for particles immersed in an anisotropic radiation field like in circumstellar shells (especially in the case of oblate grains, see Voshchinnikov & Semenov 1999 for discussion). A similar situation exists near the the edges of dark interstellar clouds. For a fixed $r_{\rm V}$ the difference in temperatures of porous spherical and non-spherical particles is smaller than for compact ones.

The temperature of interstellar dust grains can be found by fitting the galactic IR emission by modified blackbody curves. The dust emission spectrum obtained from COBE data for dust associated with H I gas can be represented by a single modified blackbody curve with $B_\nu(17.5\,{\rm K})\nu^2$ (Boulanger et al. 1996). In order to compare the observationally-based emissivity law with dust models, the shape, size, and porosity distribution of the particles have to be taken into account. It is not the goal of this paper to perform such an analysis, but to provide necessary input data. From Table 1, it can be clearly seen that the temperature of refractory spheroidal grains with $a/b = 4$ ranges between 12.1 K and 15.7 K, lower than predicted by the observations.

The steady-state temperature of grains also dictates the efficiency of the process of molecule formation on grain surfaces. As it is shown by Pirronello et al. (1999), the decrease of a grain temperature by $20 - 30\%$ can enhance the efficiency of hydrogen recombination by $2 - 4$ times.

4. Conclusions

In the conditions typical of the interstellar radiation field the temperature of the non-spherical (spheroidal) grains deviates from that of spheres of the same volume less than 10% if the aspect ratios $a/b \lesssim 2$. More elongated or flattened particles are usually cooler than spheres and in some cases the temperatures may differ by a factor 2 and more. The shape effects are almost independent of particle size but increase with the growth of the material absorption in the infrared (i.e., they are more important for carbonaceous and metallic particles than for silicates and ices). In
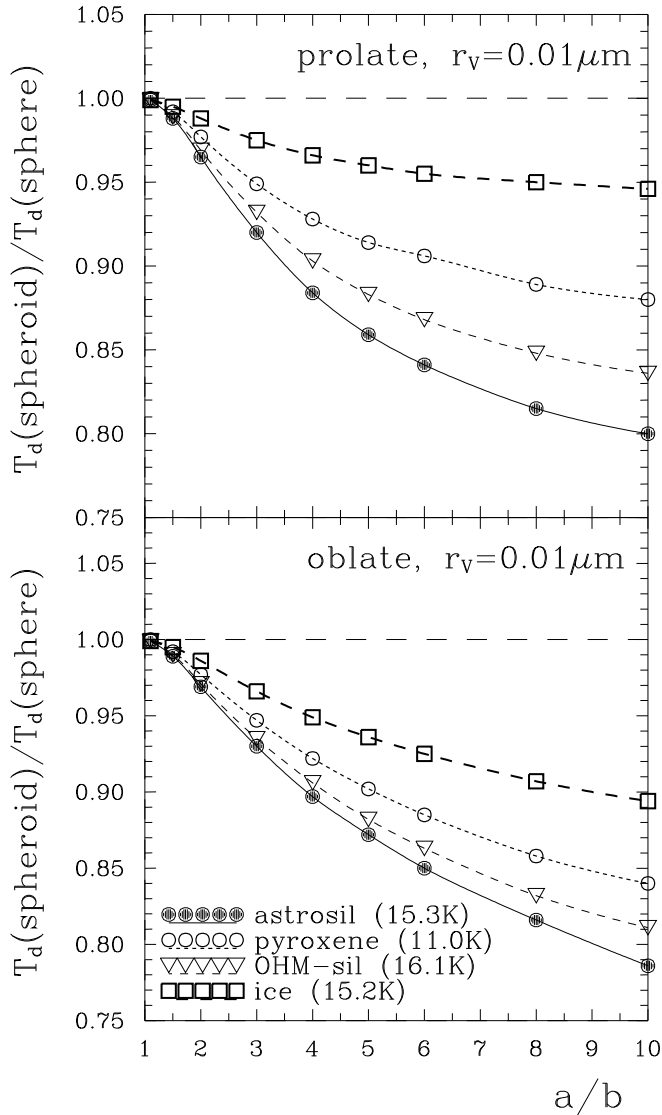
Fig. 2. The same as Fig. 1 but now for dielectric materials.

dark interstellar clouds the non-spherical particles will be cooler than spheres, facilitating the molecule formation on grain surfaces.

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